COMMUNICATIONS TO THE EDITOR

On Size Dependent Growth Rate Expressions

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Two recent papers (*1, 3*) have discussed the requirements of crystal growth expressions which are used to describe the size distributions of product crystals produced in steady, well stirred crystallization operations. The later one (*1*) lists the properties desirable for a growth model so that is may be generally useful when incorporated into the population distribution equation (*7*). The motivation for these studies was the well justified desire to treat, with population balance methods, experimental distributions which were not in accord with distribution behavior suggested by McCabe's ΔL law.

The population balance notion represents a powerful analytical device and deserves the emphasis which it receives as a means for the interpretation of laboratory and industrial crystallizer data. Distribution data reflect not only growth behavior, but nonuniformity of agitation and classification effects. Furthermore, growth may occur due to several simultaneous mechanisms so that the processes giving rise to a particular crystal size distribution may be very complex.

This note is written to suggest that to describe these complicated phenomena by means of a single growth expression of specified form which is analytic over the size range $0 < L < \infty$ is perhaps too demanding of that expression. This is especially true when the expression attempts to reflect the physical mechanisms involved.

It is recommended here that the division of sizes into intervals associated with appropriate mechanisms is straightforward and lends itself readily for use in data interpretation and scale-up.

As an illustration of the above statements the data provided in (*1, 3*) are shown in Figures 1 and 2. These data were obtained directly from the graphs given in these references. Two lines through the data are considered to provide an acceptable fit. The lower points were subject to experimental error (*8*) and fitting these points was not judged crucial in the original references and was considered similarly here. An additional line could have been added to each graph which passed through these points if such procedure appeared justifiable. Growth expressions which give rise to the distributions illustrated are, in the notation of Randolph and Larson (*7*),

$$r = k_1 S_1^a \qquad 0 \leq L < L_0 \tag{1}$$

$$r = k_2 S_2^b \qquad L \geq L_0 \tag{2}$$

and the distribution is provided by integration of the population balance equation

$$\frac{d(rn)}{dL} + \frac{n}{T} = 0 \tag{3}$$

The general solution over a domain of definition in which r is continuous is

$$rn = (rn)_{L_c} \exp\left[-\frac{1}{T}\int_{L_c}^{L}\frac{dL}{r(L)}\right] \tag{4}$$

The distribution associated with expressions (1) and (2) and which may be considered as illustrated in Figure 1 is

$$n = n^0 \exp\left(-\frac{L}{Tk_1S_1^a}\right); \quad 0 \leq L \leq L_0 \tag{5}$$

$$n = n^0 \exp\left[\left(\frac{-L_0}{Tk_1S_1^a}\right) + \left(\frac{L_0}{Tk_2S_2^b}\right)\right] \exp\left(-\frac{L}{Tk_2S_2^b}\right); \quad L \geq L_0 \tag{6}$$

The continuity of n was required. If the discontinuity in the function of L given by growth descriptions (1) and (2) is to be avoided, (1) may be replaced by

$$r = k_1'S_1^a + \left(\frac{k_2S_2^b - k_1'S_1^a}{L_0}\right)L; \quad 0 \leq L \leq L_0 \tag{7}$$

and (2) is considered unchanged. The distribution is, for Equations (7) and (2), respectively

$$n = n^0\left(\frac{k_1'S_1^a}{k_1'S_1^a + BL}\right)^{\frac{TB+1}{TB}}; 0 \leq L \leq L_0 \tag{8}$$

where

$$B = \frac{k_2S_2^b - k_1'S_1^a}{L_0}$$

$$n = n^0\left(\frac{k_1'S_1^a}{k_2S_2^b}\right)^{\frac{TB+1}{TB}} \cdot \exp\left(\frac{L_0}{Tk_2S_2^b}\right) \cdot \exp\left(-\frac{L}{Tk_2S_2^b}\right); L \geq L_0 \tag{9}$$

This distribution is illustrated in Figure 2.

Equation (9) is exponential in the variable L — there are no mathematical difficulties associated with moments as discussed in (1). Equation (8) provides no difficulties as $L \rightarrow 0$. Had there been any third analytical expression describing the growth in a range intermediate to the two given above, mathematical difficulties could easily be avoided so long as the integrand $[r(L)]^{-1}$ is integrable between L_{0_1} and L_{0_2} (both finite).

The significant point here concerns the parameters involved, in the illustration these are k_1, k_2, L_0; these are either directly meaningful as a coefficient representing interfacial resistance (5) or a mass transfer coefficient representing diffusional resistance (a or $b = 1$) (2, 6).

L_0 may have been determined from the nature of the data serving to separate the two growth mechanisms and is thus strictly empirical; or it may have been inferred from the nature of the fluid mechanical turbulence (6). In either case it may serve as a correlating parameter for more than one run despite changed crystallizing conditions. The subscripts on the S's implying size dependent supersaturation, may come about due to temperature effects as discussed by Harriott (4).

In summary, it is suggested above that by considering appropriate mechanisms relating to growth rates of suspended crystals for corresponding size intervals, physical reality is readily involved in the formulation or interpretation of population density functions. By carefully selecting the nature of the expressions in the interval bordering on zero and in that including the largest sizes, mathematical inconsistencies may be avoided. This is in contrast to formulating growth as a single analytic expression which is simple and satisfies the mathematical requirements of a growth function when included in the density function; it is likely that the parameters involved convey little physical significance.

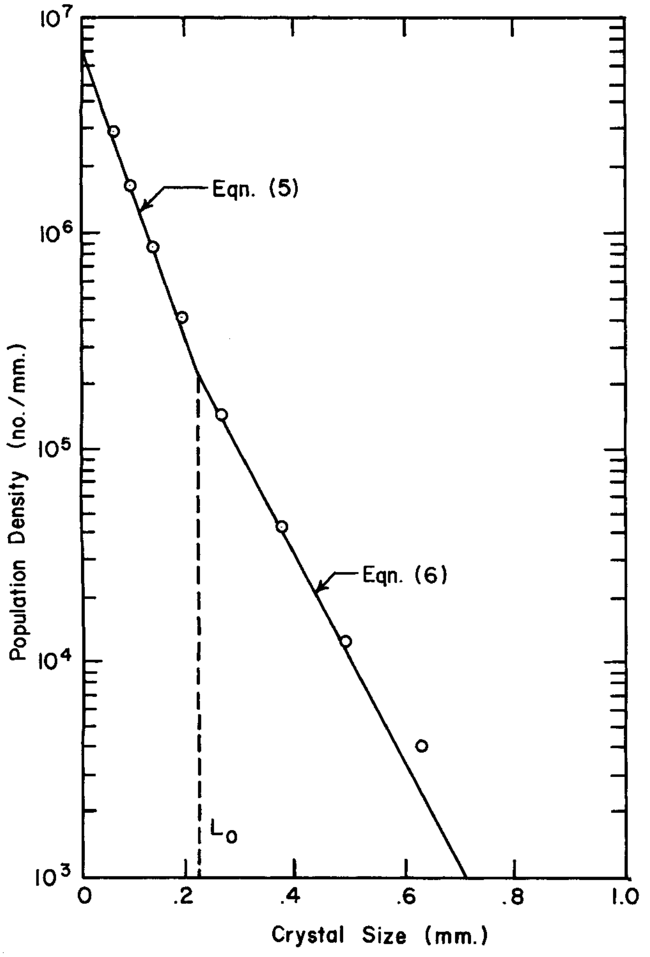

Fig. 1. Steady state size distribution data of Chambliss for Alum. Data redrawn from (1).

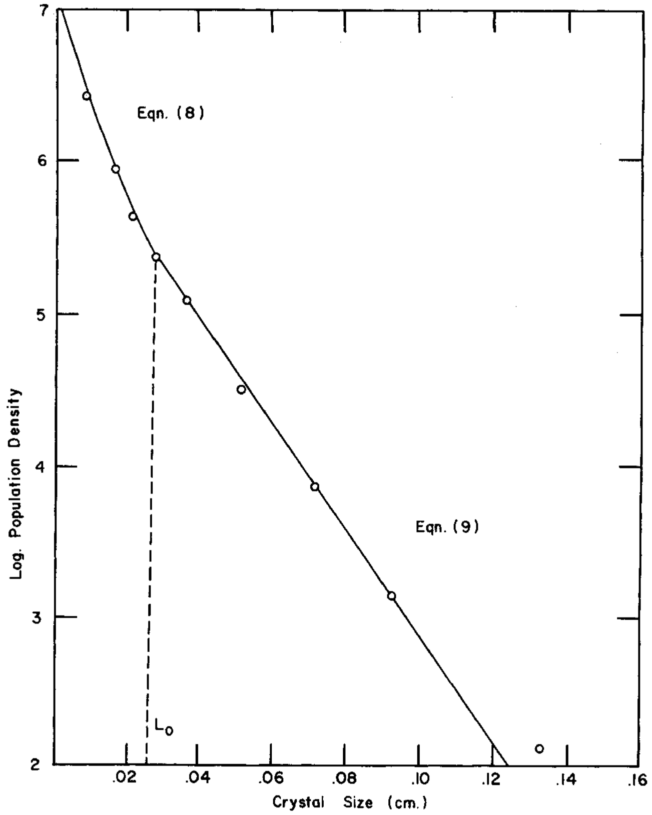

Fig. 2. Population distribution for Glauber's salt. Data redrawn from (3).

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NOTATION

a = exponent, Equations (1) or (7)
b = exponent, Equation (2)
k_1 = coefficient in growth expression, Equation (1)
k_2 = coefficient in growth expression, Equation (2)
k_1' = coefficient in growth expression, Equation (7)
L = characteristic size of crystal
L_c = characteristic size of crystal serving as lower limit to some size interval
L_0 = characteristic size of crystal serving as lower limit to large sizes in given illustration
n = density function giving number distribution of sizes
n_0 = value of density function when $L = 0$
r = linear growth rate
S = supersaturation
T = residence time

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